

Embedded Set Partition Coding

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Outline

- SPECK (Set Partition Embedded bloCK)
coding
- Features
 - Low complexity
 - Embeddedness
 - Implementation issues
- Results and Images
- Conclusions



Low Complexity Set Partitioning Embedded Coder: SPECK

- SPECK discovered by Islam and Pearlman (VCIP'99, Proc. SPIE 3653) and possibly others
 - Partitions and codes sets of coefficients grouped within subbands
- Can encode any grouping of pixels or coefficients
- Bitstream is *embedded* – lower rate image can be decoded from subset of bitstream

Partitioning of Significant Set S into Offspring

Operate through significance decisions for partitioning sets of {wavelet transform} coefficients.

Sets and partitioning rules differ between SPECK and SPIHT.

Significance test for pixels $c_{i,j}$ in set S

$$S_n(S) = \begin{cases} 1, & \text{if } \left(\max_{(i,j) \in S} |c_{ij}| \right) \geq 2^n, \\ 0, & \text{otherwise.} \end{cases}$$

$O_0(S)$	$O_1(S)$
$O_2(S)$	$O_3(S)$

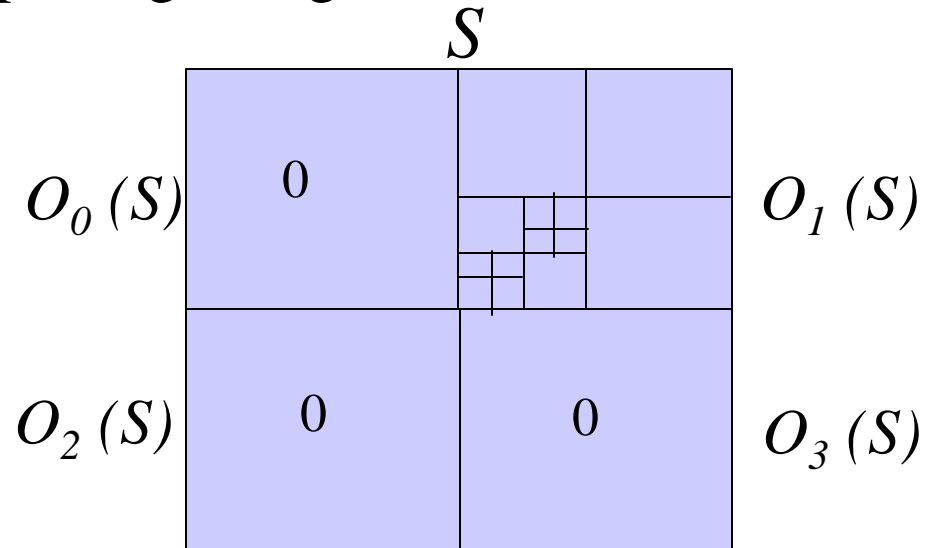
Set Partitioning Rules of SPECK

S Partitioned into Offspring Sets

Recursive quadrature splitting of significant sets

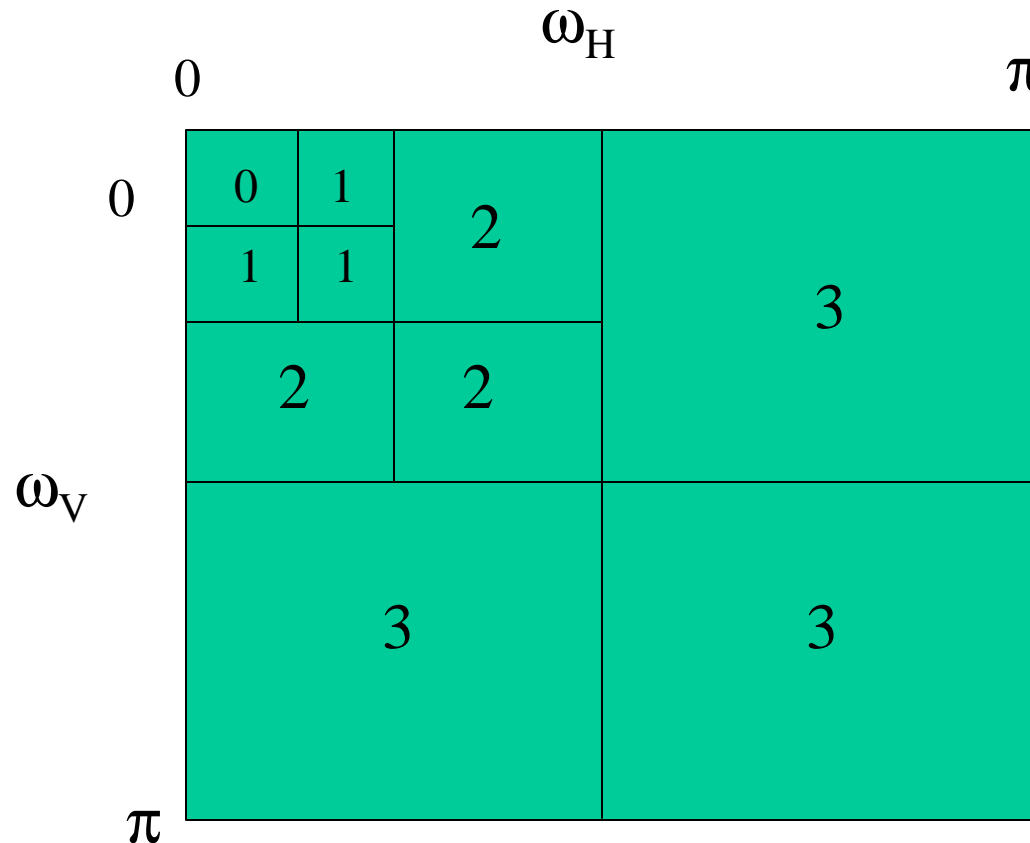
LIS - list of insignificant sets in increasing order of size starting with single element sets

LSP - list of significant single elements.



For given threshold or bit plane n , insignificant sets and coefficients go to LIS, significant coefficients to LSP.
Decrease n by 1, repeat.

Multi-resolution (Wavelet) Transform

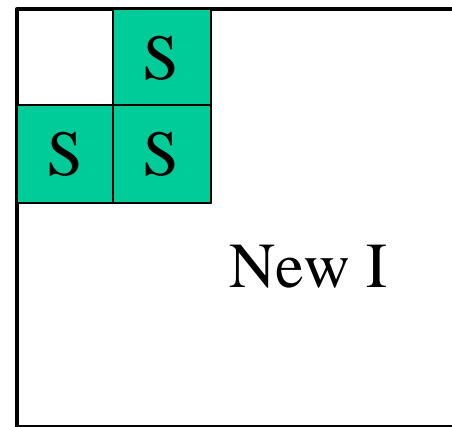
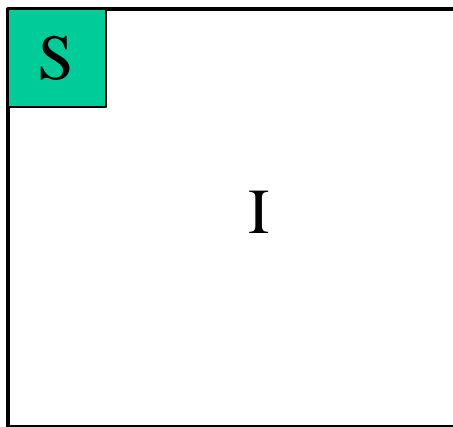



0 -> 1/8 res.
 +
 1,1,1 -> 1/4 res.
 +
 2,2,2 -> 1/2 res.
 +
 3,3,3 -> full res.

Octave Band Partitioning

Test through full transform for each n ,

$$n = n_{\max}, n_{\max} - 1, n_{\max} - 2, \dots \quad n_{\max} = \max_{i,j} \lfloor \log_2 |c_{i,j}| \rfloor$$

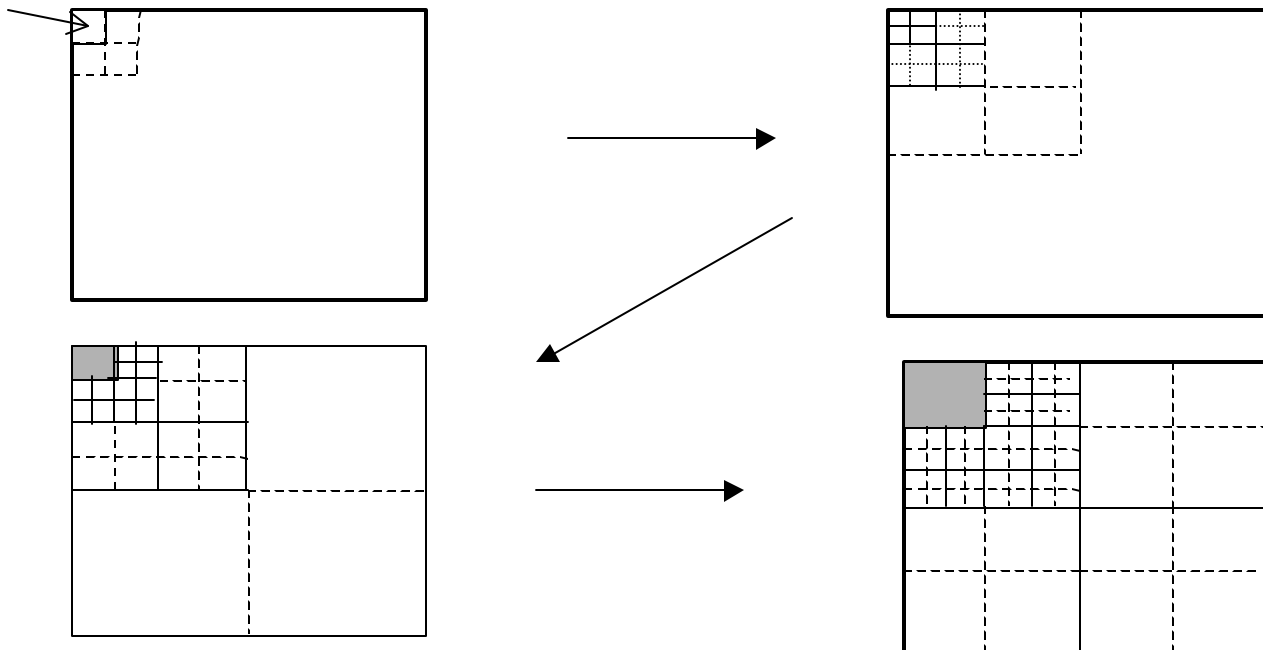


 is coded

Subband Partitioning Order

- Partitioning proceeds from coarse to fine scale

Initial S



Coding Sequence

- SPECK uses two lists LIS and LSP visited for significance testing in that order, for a given bit-plane (n).
 - LIS: list of coordinates of insignificant coefficients organized into sublists of increasing size from single element sets at the top
 - Functionally equivalent to separate LIP list
 - LSP: list of coordinates of significant pixels
 - Send n -th bit of all pixels found to be significant in previous passes.

LSP Sorting by Magnitude and Progressive Bit-Plane Transmission

Transmission of magnitude-sorted coefficients

	sign	S	S	S	S	S	S	S	S	S	S	S	S	S
msb	5	1	1	0	0	0	0	0	0	0	0	0	0	0
	4	Ⓜ	Ⓜ	1	1	0	0	0	0	0	0	0	0	0
	3	Ⓜ	Ⓜ	Ⓜ	Ⓜ	1	1	1	1	0	0	0	0	0
	2	Ⓜ	Ⓜ	Ⓜ	Ⓜ	Ⓜ	Ⓜ	Ⓜ	Ⓜ	1	1	1	1	1
	1	Ⓜ	Ⓜ	Ⓜ	Ⓜ	Ⓜ	Ⓜ	Ⓜ	Ⓜ	Ⓜ	Ⓜ	Ⓜ	Ⓜ	Ⓜ
lsb	0	Ⓜ	Ⓜ	Ⓜ	Ⓜ	Ⓜ	Ⓜ	Ⓜ	Ⓜ	Ⓜ	Ⓜ	Ⓜ	Ⓜ	Ⓜ

Send n -th bit of all coefficients found to be significant in previous passes.

Transmission of Bits

- LSP sign and refinement bits sent raw
 - Possible to encode by context-based arithmetic coding to achieve small gains
- Significance decision encoded or sent raw
 - Context-based arithmetic coding in 2x2 sets achieves some gains
 - Most entropy-coding gain can be obtained with fixed or semi-adaptive Huffman codes
- Fully adaptive entropy-coded version of SPECK called EZBC surpasses JPEG2000

Pseudo-Code of SPECK Algorithm

1. Initialization

- Partition image transform \mathcal{X} into two sets: $\mathcal{S} \equiv \text{root}$, and $\mathcal{I} \equiv \mathcal{X} - \mathcal{S}$
- output $n = \lfloor \log_2(\max_{\forall(i,j) \in \mathcal{X}} |c_{i,j}|) \rfloor$
- add \mathcal{S} to LIS and set $\text{LSP} = \phi$

2. Sorting Pass

- in increasing order of size \mathcal{C} of sets

– for each set $\mathcal{S} \in \text{LIS}$,

* $\text{ProcessS}(\mathcal{S})$

● $\text{ProcessI}()$

3. Refinement Pass

● for each $(i, j) \in \text{LSP}$, except those included in the last sorting pass, output the n th MSB of $|c_{i,j}|$

4. Quantization Step

● decrement n by 1, and go to step 2

Process $\mathcal{S}(\mathcal{S})$ {

- output $\mathcal{S}_n(\mathcal{S})$
- if $\mathcal{S}_n(\mathcal{S}) = 1$
 - if \mathcal{S} is a coefficient, output sign of \mathcal{S} and add \mathcal{S} to LSP
 - else Code $\mathcal{S}(\mathcal{S})$
 - if $\mathcal{S} \in \text{LIS}$, remove \mathcal{S} from LIS
- else

– if $S \notin \text{LIS}$, add S to LIS } }

CodeS(\mathcal{S}) {

- Partition \mathcal{S} into four equal subsets $\mathcal{O}(\mathcal{S})$
- for each $\mathcal{O}(\mathcal{S})$
 - output $\mathcal{S}_n(\mathcal{O}(\mathcal{S}))$
 - if $\mathcal{S}_n(\mathcal{O}(\mathcal{S})) = 1$
 - * if $\mathcal{O}(\mathcal{S})$ is a coefficient, output sign of $\mathcal{O}(\mathcal{S})$ and add $\mathcal{O}(\mathcal{S})$ to LSP

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* else CodeS( $\mathcal{O}(\mathcal{S})$ )  
- else  
* add  $\mathcal{O}(\mathcal{S})$  to LIS }
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ProcessI() {
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- output $\mathcal{S}_n(\mathcal{I})$
- if $\mathcal{S}_n(\mathcal{I}) = 1$
 - CodeI() }

CodeI() {

- Partition \mathcal{I} into four sets — three \mathcal{S} and one \mathcal{I}

- for each of the three sets \mathcal{S}

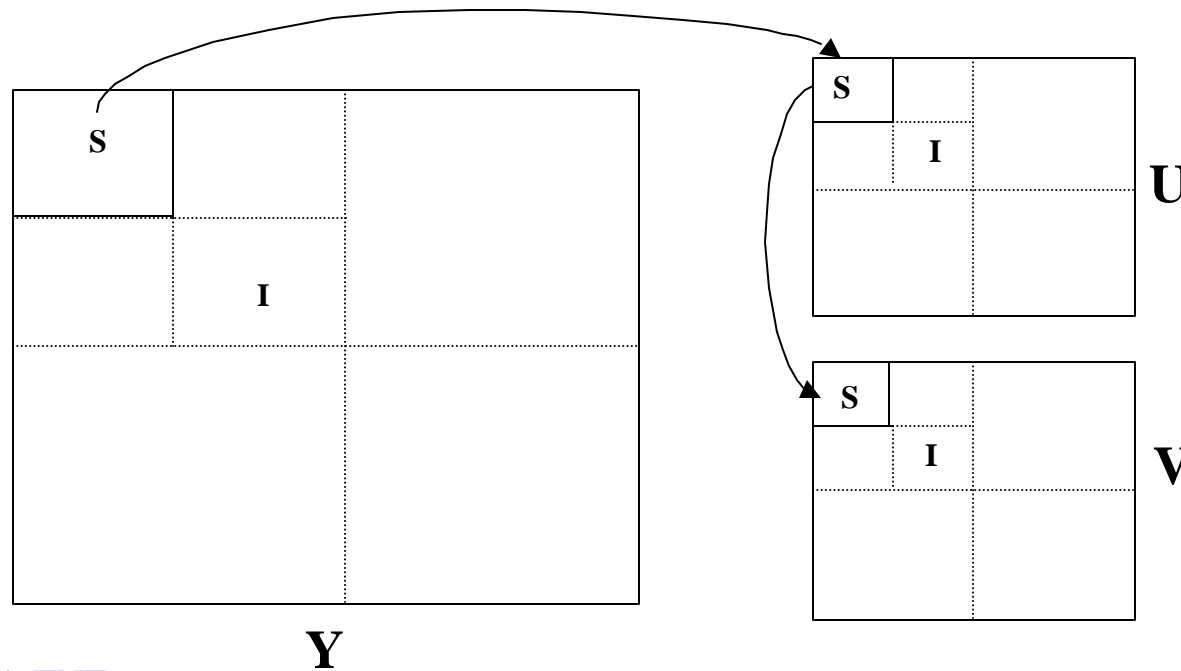
– ProcessS(\mathcal{S})

- ProcessI() }

Comment	Point or Set Tested	Output Bit	Action	Control Lists
n=5 Sorting				LIS = $\{(0,0)\mathbf{1}\}$ LSP = ϕ
	$S^1(0,0)$	1	quad split, add to LIS(0)	LIS = $\{(0,0)\mathbf{0}, (0,1)\mathbf{0}, (0,1)\mathbf{0}, (1,0)\mathbf{0}, (1,1)\mathbf{0}\}$ LSP = ϕ
	(0,0)	1+	(0,0) to LSP	LIS = $\{(0,1)\mathbf{0}, (0,1)\mathbf{0}, (1,0)\mathbf{0}, (1,1)\mathbf{0}\}$ LSP = $\{(0,0)\}$
	(0,1)	1-	(0,1) to LSP	LIS = $\{(1,0)\mathbf{0}, (1,1)\mathbf{0}\}$ LSP = $\{(0,0), (0,1)\}$
	(1,0)	0	none	
	(1,1)	0	none	
	$S^1(0,2)$	1	quad split, add to LIS(0)	LIS = $\{(1,0)\mathbf{0}, (1,1)\mathbf{0}\}, (0,2)\mathbf{0}, (0,3)\mathbf{0}\}$ $(1,2)\mathbf{0}, (1,3)\mathbf{0}\}$
	(0,2)	1+	(0,2) to LSP	LSP = $\{(0,0), (0,1), (0,2)\}$ LIS = $\{(1,0)\mathbf{0}, (1,1)\mathbf{0}\}, (0,3)\mathbf{0}\}$ $(1,2)\mathbf{0}, (1,3)\mathbf{0}\}$
	(0,3)	0	none	
	(1,2)	0	none	
	(1,3)	0	none	
	$S^1(2,0)$	0	add to LIS(1)	LIS = $\{(1,0)\mathbf{0}, (1,1)\mathbf{0}\}, (0,3)\mathbf{0}\}$ $(1,2)\mathbf{0}, (1,3)\mathbf{0}, (2,0)\mathbf{1}\}$
	$S^1(2,3)$	0	add to LIS(1)	LIS = $\{(1,0)\mathbf{0}, (1,1)\mathbf{0}\}, (0,3)\mathbf{0}\}$ $(1,2)\mathbf{0}, (1,3)\mathbf{0}, (2,0)\mathbf{1}, (2,3)\mathbf{1}\}$
	$S^2(0,4)$	0	add to LIS(1)	LIS = $\{(1,0)\mathbf{0}, (1,1)\mathbf{0}\}, (0,3)\mathbf{0}\}$ $(1,2)\mathbf{0}, (1,3)\mathbf{0}, (2,0)\mathbf{1}, (2,3)\mathbf{1}, (0,4)\mathbf{2}\}$
	$S^2(4,0)$	1	quad split, add to LIS(1)	LIS = $\{(1,0)\mathbf{0}, (1,1)\mathbf{0}, (0,3)\mathbf{0}, (1,2)\mathbf{0}, (1,3)\mathbf{0}, (2,0)\mathbf{1}, (2,3)\mathbf{1}, (4,0)\mathbf{1}, (4,2)\mathbf{1}, (6,0)\mathbf{1}, (6,2)\mathbf{1}, (0,4)\mathbf{2}\}$
	$S^1(4,0)$	0	none	
	$S^1(4,2)$	1	quad split, add to LIS(0)	LIS = $\{(1,0)\mathbf{0}, (1,1)\mathbf{0}, (0,3)\mathbf{0}, (1,2)\mathbf{0}, (1,3)\mathbf{0}, (4,2)\mathbf{0}, (4,3)\mathbf{0}, (5,2)\mathbf{0}, (5,3)\mathbf{0}, (2,0)\mathbf{1}, (2,3)\mathbf{1}, (4,0)\mathbf{1}, (4,2)\mathbf{1}, (6,0)\mathbf{1}, (6,2)\mathbf{1}, (0,4)\mathbf{2}\}$
	(4,2)	0	none	
	(4,3)	1+	move (4,3) to LSP	LSP = $\{(0,0), (0,1), (0,2), (4,3)\}$ LIS = $\{(1,0)\mathbf{0}, (1,1)\mathbf{0}, (0,3)\mathbf{0}\}$ $(1,2)\mathbf{0}, (1,3)\mathbf{0}, (4,2)\mathbf{0}, (5,2)\mathbf{0}, (5,3)\mathbf{0}, (2,0)\mathbf{1}, (2,3)\mathbf{1}, (4,0)\mathbf{1}, (4,2)\mathbf{1}, (6,0)\mathbf{1}, (6,2)\mathbf{1}, (0,4)\mathbf{2}\}$
	(5,2)	0	none	
	(5,3)	0	none	
	$S^1(6,0)$	0	none	
	$S^1(6,2)$	0	none	
End n=5 Sorting	$S^2(4,4)$	0	add to LIS(2)	LIS = $\{(1,0)\mathbf{0}, (1,1)\mathbf{0}, (0,3)\mathbf{0}, (1,2)\mathbf{0}, (1,3)\mathbf{0}, (4,2)\mathbf{0}, (5,2)\mathbf{0}, (5,3)\mathbf{0}, (2,0)\mathbf{1}, (2,3)\mathbf{1}, (4,0)\mathbf{1}, (4,2)\mathbf{1}, (6,0)\mathbf{1}, (6,2)\mathbf{1}, (0,4)\mathbf{2}, (0,4)\mathbf{2}\}$ LSP = $\{(0,0), (0,1), (0,2), (4,3)\}$

Comment	Point or Set Tested	Output Bit	Action	Control Lists
n=4 Sorting				LIS = $\{(1,0)\mathbf{0},(1,1)\mathbf{0},(0,3)\mathbf{0},(1,2)\mathbf{0},(1,3)\mathbf{0},(4,2)\mathbf{0},(5,2)\mathbf{0},(5,3)\mathbf{0},(2,0)\mathbf{1},(2,3)\mathbf{1},(4,0)\mathbf{1},(4,2)\mathbf{1},(6,0)\mathbf{1},(6,2)\mathbf{1},(0,4)\mathbf{2},(0,4)\mathbf{2}\}$ LSP = $\{(0,0),(0,1),(0,2),(4,3)\}$
Test LIS(0)	(1,0)	1-	(1,0) to LSP	
	(1,1)	1+	(1,1) to LSP	LIS = $\{(0,3)\mathbf{0},(1,2)\mathbf{0},(1,3)\mathbf{0},(4,2)\mathbf{0},(5,2)\mathbf{0},(5,3)\mathbf{0},(2,0)\mathbf{1},(2,3)\mathbf{1},(4,0)\mathbf{1},(4,2)\mathbf{1},(6,0)\mathbf{1},(6,2)\mathbf{1},(0,4)\mathbf{2},(0,4)\mathbf{2}\}$ LSP = $\{(0,0),(0,1),(0,2),(4,3),(1,0),(1,1)\}$
	(0,3)	0	none	
	(1,2)	0	none	
	(1,3)	0	none	
	(4,2)	0	none	
	(5,2)	0	none	
	(5,3)	0	none	
Test LIS(1)	$S^1(2,0)$	0	none	
	$S^1(2,3)$	0	none	
	$S^1(4,0)$	0	none	
	$S^1(6,0)$	0	none	
	$S^1(6,2)$	0	none	
Test LIS(2)	$S^2(0,4)$	0	none	
	$S^2(4,4)$	0	none	
Refinement	(0,0)	1	decoder adds 2^4	
	(0,1)	0	decoder subtracts 0	
	((0,2)	1	decoder adds 2^4	
	(4,3)	0	decoder adds 0	
End n=4				

- For a given threshold; the luminance component is first divided into an 'S' and an 'I' set (see figure) and processed.
- Following this, the chrominance components are divided into an 'S' and an 'I' set and processed *at the same threshold*.
- The threshold is reduced by half (bit-plane coding) after the refinement pass and the algorithm repeats.



Comparison of various color codecs

KBits	Y/C	JPEG 2000	SPIHT	CSPECK	MPEG-4
20.9	Y	34.65	35.40	35.83	35.09
(Hall)	C	41.27	39.73	37.62	39.78
28.5	Y	37.73	38.05	38.50	37.82
(Hall)	C	43.14	40.71	39.79	41.16
19.3	Y	33.65	34.33	34.77	33.95
(foreman)	C	42.54	40.32	37.42	40.56
28.7	Y	37.42	37.53	37.71	37.11
(foreman)	C	43.70	43.04	40.61	42.08

these figures indicate PSNR (in dB)

Original Image "Foreman.QCIF"



CSPECK (19.3 Kbits)



SPIHT (19.3 Kbits)



JPEG2000 (19.3 Kbits)



Motion-SPECK vs. Motion-JPEG2000

Codec	Mean PSNR in dB			Decoding Times * (sec)
	Y	U	V	
<i>“Carphone” (97 frames) at 0.5 bits/pixel/frame</i>				
Motion-JPEG2000	31.23	38.28	39.17	8.43
Motion-SPECK	32.56	35.97	36.25	8.74
<i>“Trevor” (150 frames) at 0.25 bits/pixel/frame</i>				
Motion-JPEG2000	28.59	36.73	37.28	11.39
Motion-SPECK	30.59	34.26	35.14	12.10

*Decoding time is based on Sun SPARC 10



(a)



(b)



(c)

- (a) Original image, frame no. 77 of “Carphone” sequence
- (b) Motion-SPECK reconstructed (0.5bpp/frame)
- (c) Motion-JPEG2000 reconstructed (0.5bpp/frame)

Embedding Issues

- Full transform SPECK (and SPIHT) fully bit-embedded
 - next bit conveys less value information than previous one
- Full bit-embedding costs in speed and complexity
 - many applications do not require full bit-embedding and/or can not afford it (viz., geographic images, real-time animation)
- SPECK (and SPIHT) allow relaxation to *value embedding* by skipping refinement pass and sending all lower bit planes immediately for each new LSP coefficient.
 - *can not be done in JPEG2000*

Small Memory Implementations

- SPECK used in small subband blocks (SBHP)
 - replaced coding of subband blocks in JPEG2000 framework
- SPIHT implemented in JPEG2000 framework
 - SPIHT coding applied to small transform tree-blocks independently
 - Same could be done with SPECK
- Embedded bit streams from independent tree-block or subband block encoders are packetized and multiplexed to generate a progressive main bit stream

- Wavelet transform and set partition coding bring many desirable features:
 - Scalability in resolution and fidelity
 - Low complexity in computation and memory
 - Fast encoding and decoding
- Key to low complexity
 - set partitioning and testing for set significance
 - Adaptive techniques with higher complexity
 - SLCC (Chai et al), PACC (Marpe & Cycon), GTW (Hong & Ladner)
- Other features
 - Region of interest (ROI) fidelity enhancement
 - Error resilient bitstream
 - Parallelizable for embedded hardware applications