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# Milestones and Trends in Image Compression

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# Outline

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- Key historical developments
- Where are we?
- Trends. Improvements? Breakthroughs?
- Conclusions

# Beginnings of Image Coding

- Shannon 1948:
  - entropy is lowest bit rate possible for perfect recovery
- Shannon 1960:
  - Rate-distortion function ( $R(D)$ ) gives lowest bit rate possible for reconstruction with distortion no greater than  $D$



# Considerations

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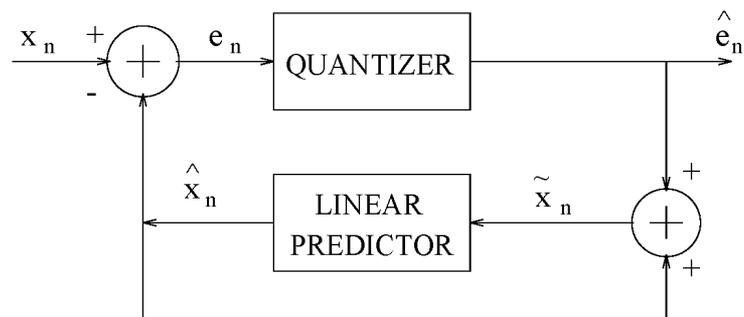
- Theorems true for statistically stationary processes – *images not stationary*
- Optimal in limit of long data length
  - not practical
- Early image coding techniques built for stationary models
  - Often simple Gaussian or Laplacian models
  - Parameters may vary block-wise or region-wise

# Early Post-Shannon Image Coding

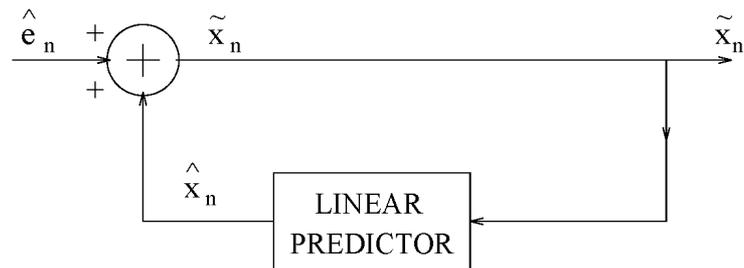
Good reviews: Netravali, Limb, Proc. IEEE 03/80;  
Connor, Brainard, Limb, Proc. IEEE 07/72

- DPCM : Prediction and coding from past quantized residuals
  - Quantization tailored to visual perception
  - Predictor from simple to adaptive

MIT (Schreiber, Proc IEEE 3/67),  
Bell Labs (Kretzmer, Limb, etc.)



(a) DPCM Encoder



(a) DPCM Decoder

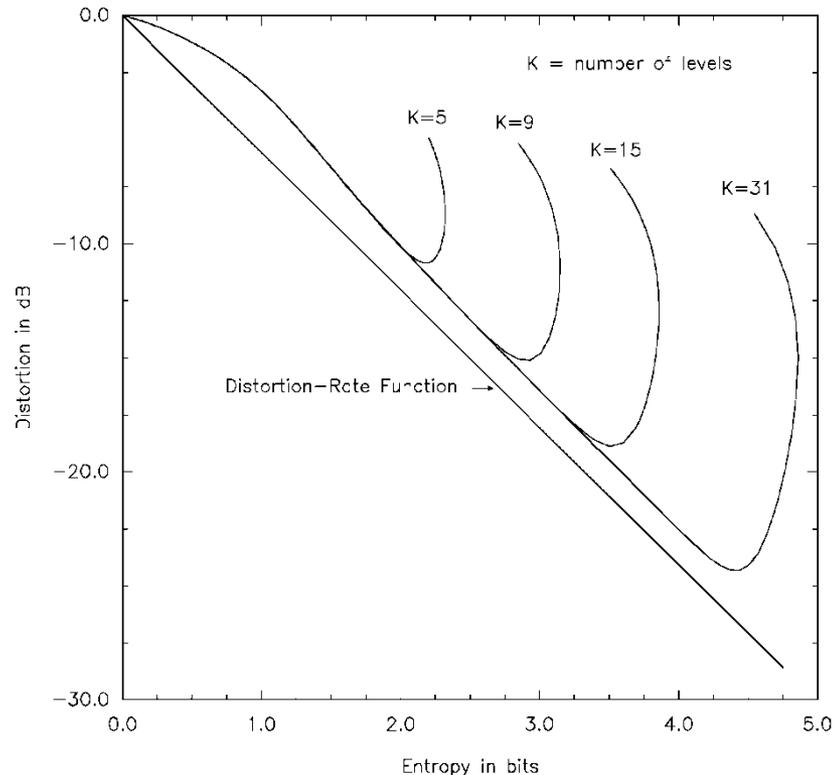
# Advent of Transform Coding

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- Huang and Schultheiss (IEEE Trans Commun. Tech. (T-CT) 1963)
  - Proved coding gain for correlated Gaussian sequence via optimal bit allocation to KLT
  - Sparked image transform coding research:
    - Purdue: Habibi and Wintz (T-CT1971), Wintz (Proc. IEEE 1972)
    - USC: Pratt and Andrews(1968-9), Chen and Pratt (T-COM 1974)
    - MIT: Anderson and Huang (T-CT '71), Woods and Huang (Picture Bandwidth Comp Wkshp 1969)

# Interesting Developments

- Goblick and Holsinger (T-IT 4/67)
  - Entropy coding of outputs of uniform quantizer nearly optimum:  
 $\frac{1}{4}$  bit  $>$   $R(D)$  for Gaussian, MSE
  - Proved more formally for other statistics by Gish and Pierce (T-IT 9/68)
- ❖ Validates uniform quantizer as choice for minimum MSE with given entropy.



# Transforms

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- Search for easily computable, fixed transforms
  - KLT optimal: dependent on statistics and no fast alg.
  - DFT asymptotically optimal, fast algorithm
  - Hadamard fastest to compute, but inefficient in coding
  - Slant, SVC harder to compute, more efficient in coding
- Applied to 16x16 image blocks
- DCT : Ahmed, Natarajan, Rao (T-Cmptr 01/74)
  - Approached KLT spectrum closely for finite N
  - Fixed, independent of statistics, with fast algorithm
  - Became dominant, canonical transform

# Region Adaptive Transform Coding

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- Chen and Pratt, “Scene Adaptive Coder”, T-Comm 1984
  - Divided 16x16 DCT blocks into 4 classes, calculated 4 intra-class variance distributions for rate allocation
  - Forerunner of JPEG standard
- JPEG Standard 1989-1992
  - Codes 8x8 DCT blocks independently with Huffman coding of uniform step size quantizer outputs
  - Huffman code based on statistics gathered from experiments with a large number of images

# Vector Quantization

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- Generalized Lloyd or LBG (Linde-Buzo-Gray) algorithm (T-COM, 01/80)
  - Asymptotically optimal: complexity  $\sim 2^{nR}$
  - Restriction to small  $n$  and statistical mismatch limited performance
- TCQ (Trellis Coded Quantization) (Marcellin & Fischer, T-COM, 1/90)
  - Asymptotically optimal: complexity  $\sim n$
  - Within 0.21 dB of  $R(D)$  for Gauss iid source ( $>1$  b/s)
  - Deteriorates in performance  $< 1$  b/s
  - Adopted in JP2000, Part II for Wavelet TCQ

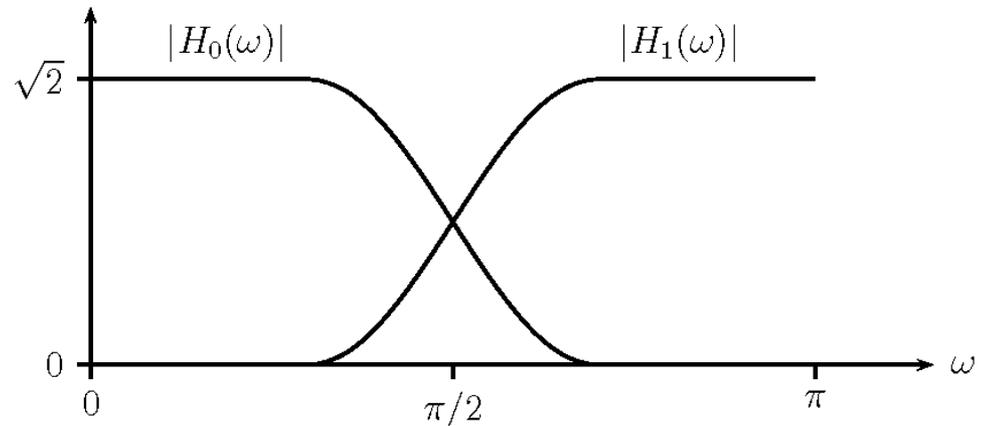
# Subband Coding

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- Woods and O'Neil (ICASSP 4/86, T-ASSP 10/86); Gharavi and Tabatabai (VCIP 10/86)
  - First subband coding of images, although done before for speech
  - DPCM coding in subbands
  - Superior results over DCT coding
    - Surprising performance – theoretically unjustified by W&O analysis, eventually justified by P. Rao, S. Rao, and Pearlman (T-IT 3/91, 7/96)

# Wavelets

- Need alias-cancelling half-band filters (a low-pass and a high-pass) for perfect reconstruction
- QMF and paraunitary filters were exact or approximate solutions



- Then came wavelet filters, specifically the CDF 9/7 biorthogonal wavelet filter
  - used first for image coding by Antonini, Barlaud *et al.* (T-IP 4/92)

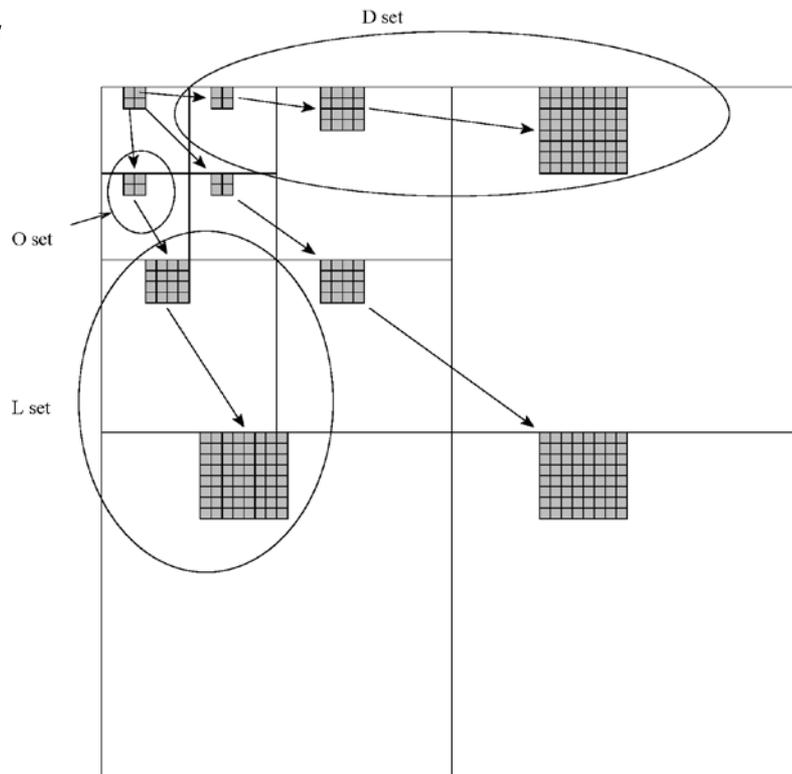
# Coding of Subbands

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- Kinds of coding of subbands independently
  - Huffman coding, arithmetic coding, predictive coding (DPCM), tree coding, trellis coding, VQ, TCQ
    - All did well, the more complex the better the result
- Significant breakthrough: *zerotree coding*
  - EZW (Shapiro, ICASSP'92, T-ASSP 12/93)
    - Takes advantage of decaying amplitude with wavelet subband frequency

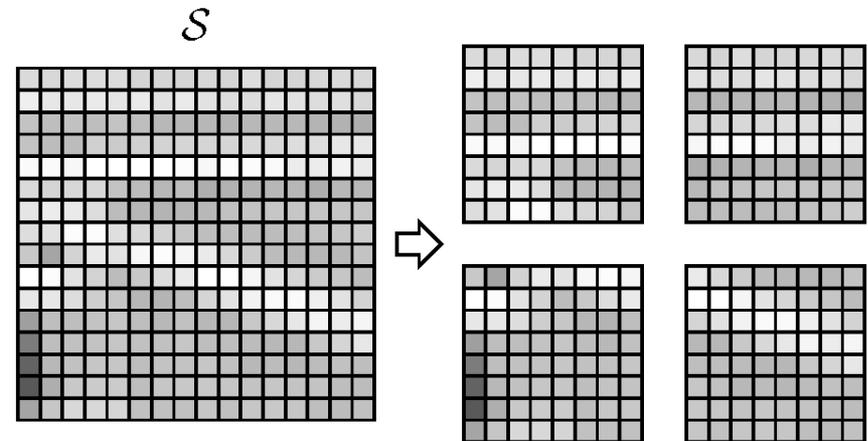
# SPIHT

- SPIHT (Said and Pearlman, T-CSVT 6/96)
  - Introduces set partitioning in spatial orientation trees with roots in lowest frequency subband
  - Finds groups of pixels below set of thresholds  $T=2^n$ 
    - reduces to  $n$  raw bits for smallest  $n$  (+ sign bit)
    - $n=0$ : 1 '0' bit locates group
- ❖ Amplitude-based, non-statistical bit assignment
- ❖ Simple arithmetic operations
- ❖ More efficient than EZW



# Another Partition (SPECK)

- SPECK (Islam and Pearlman, VCIP99)
  - Recursive quadri-section of blocks
  - Quadtree code for execution path



# JPEG2000

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- Seek better low rate performance than JPEG
  - Eliminate blocking artifacts found in low-rate JPEG
    - LOT (Malvar, 1992) was one solution
- Embed new features: rate scalability, ROI encoding/decoding, etc. (inherent in SPIHT and EZW)
- Codes subblocks of wavelet subbands with EBCOT coder (Taubman, T-IP 07/00)
  - Subblocks' size 64x64 or 32x32
  - Context-based adaptive arithmetic bitplane coding
- Part 1 finalized in 2001

# Trends

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- Get closer to code based on actual value
  - SPIHT does it almost perfectly
- Smaller coding units
  - Enables finer resolution, locally adaptive coding
    - 8x8 DCT in JPEG, 4x4 DCT in H.264/AVC, JPEG XR
    - Subband subblock coding in JPEG2000
- Overlapped blocks or inter-block prediction
  - Eliminates discontinuities at block boundaries
- More complex, adaptive context-based entropy coding (e.g., JPEG2000, H.264/AVC)
- Simpler block transforms (integerized DCT)
  - Less decorrelation compensated by more complex coding

# Efficiency of Modern Methods

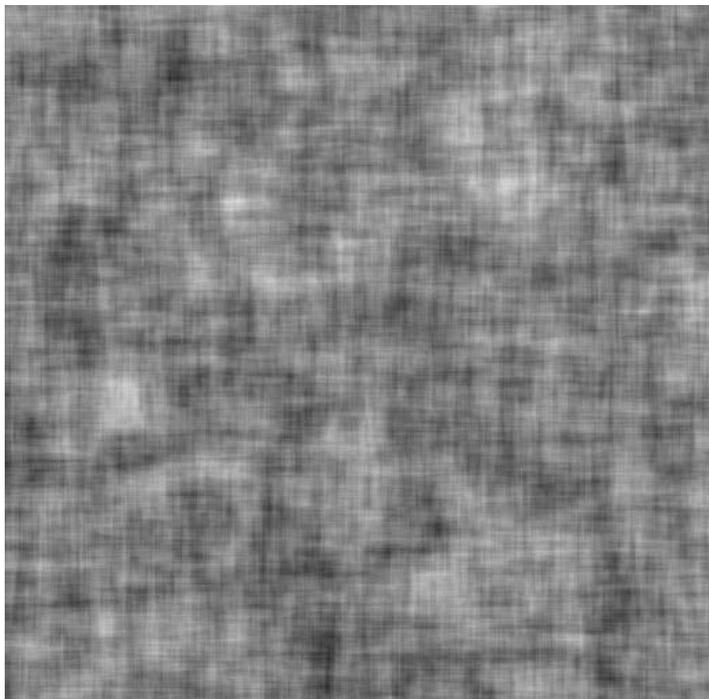
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- Methodology
  - Generate Gauss-Markov Images
  - Compare compression results with Rate-Distortion or joint entropy function

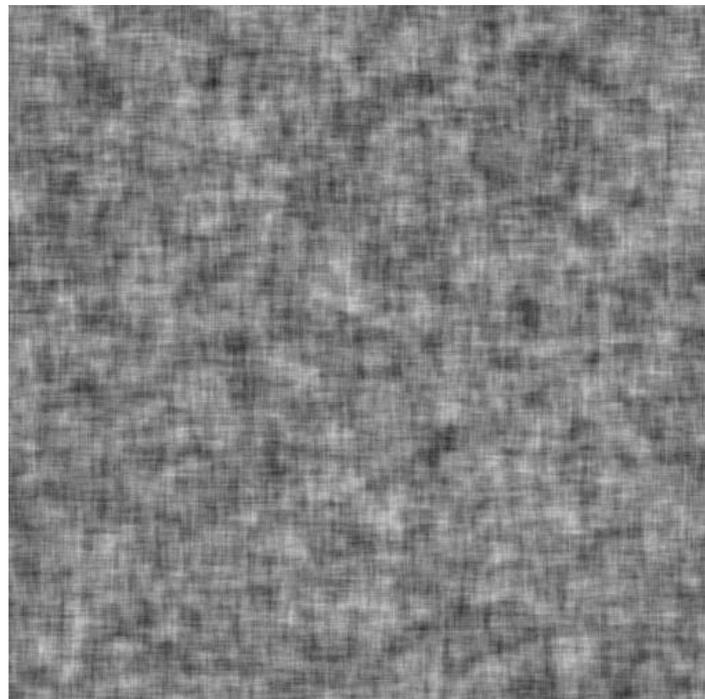
# Gauss-Markov Images

Variance = 400

Mean = 128



$a = 0.95$



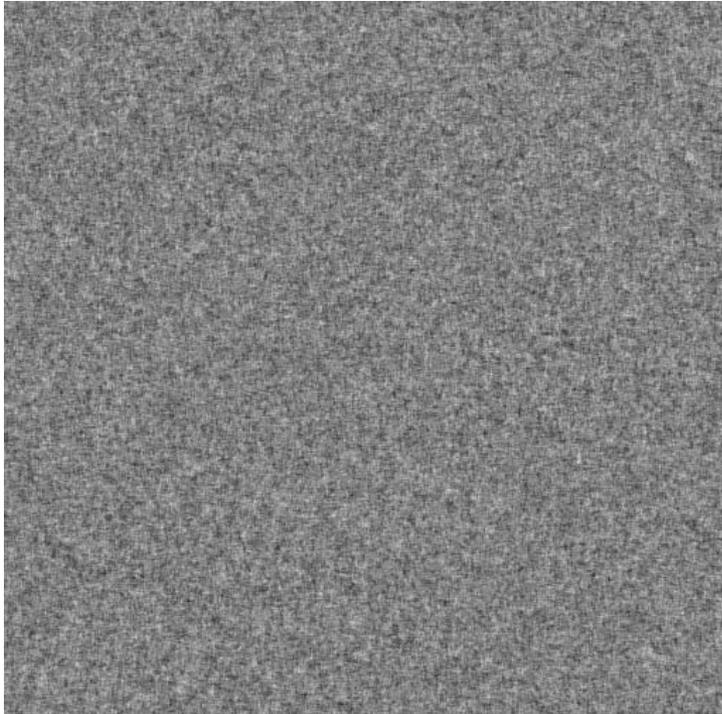
$a = 0.90$

Separable; 8-bit precision;  
512x512 lower cut from 640x640

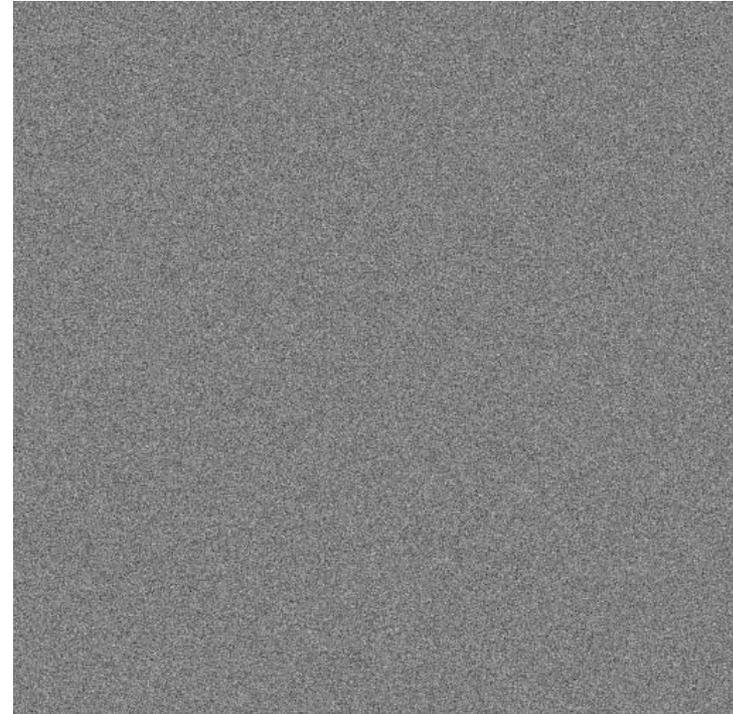
# Gauss-Markov Images (cont.)

Variance = 400

Mean = 128



$a = 0.50$



$a = 0.0$

Separable; 8-bit precision;  
512x512 lower cut from 640x640

# Theoretical Bounds

Rate-Distortion Function (Gaussian, squared error)

$$R = \frac{1}{N^2} \sum_{i=1}^N \sum_{j=1}^N \max\left\{0, \frac{1}{2} \log_2 \frac{\lambda(i)\lambda(j)}{\theta}\right\}$$

$$D = \frac{1}{N^2} \sum_{i=1}^N \sum_{j=1}^N \min\{\theta, \lambda(i)\lambda(j)\}$$

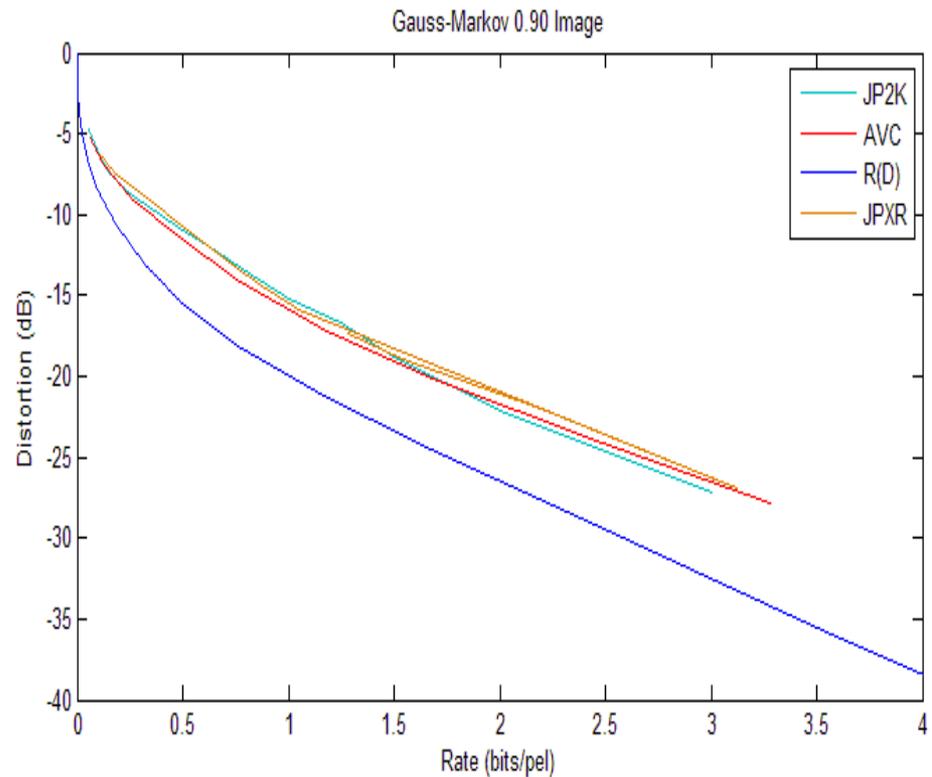
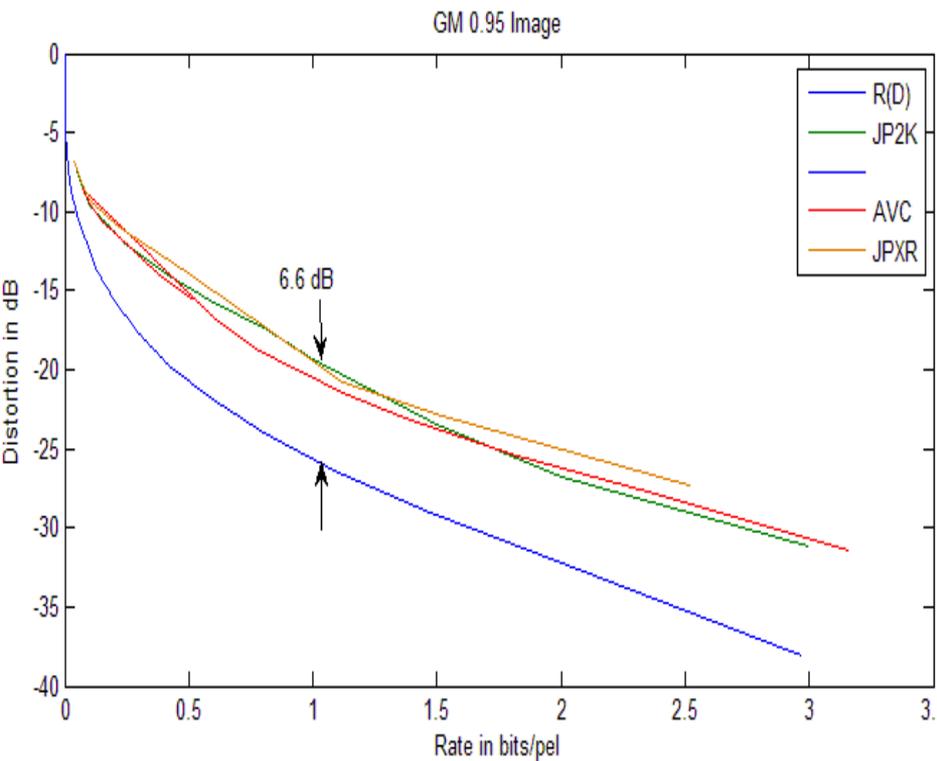
## Approximate Gaussian Entropy Function

Using  $H(X^n) \approx h(X^n) - \log_2 \delta$        $-\log_2 \delta = m$  bits (precision)

$$\frac{1}{N^2} H(X^{N^2}) \approx \frac{1}{2N^2} \sum_{i=1}^N \sum_{j=1}^N \log_2(\lambda(i)\lambda(j)) + \frac{1}{2} \log_2 2\pi e \sigma_X^2 + \frac{m}{N^2}$$

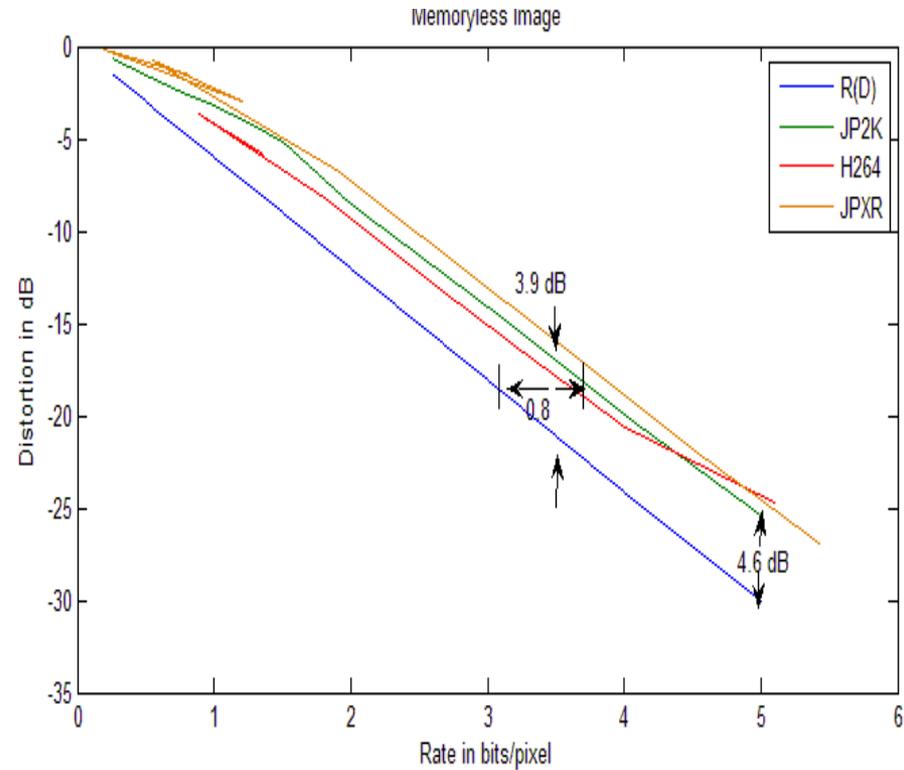
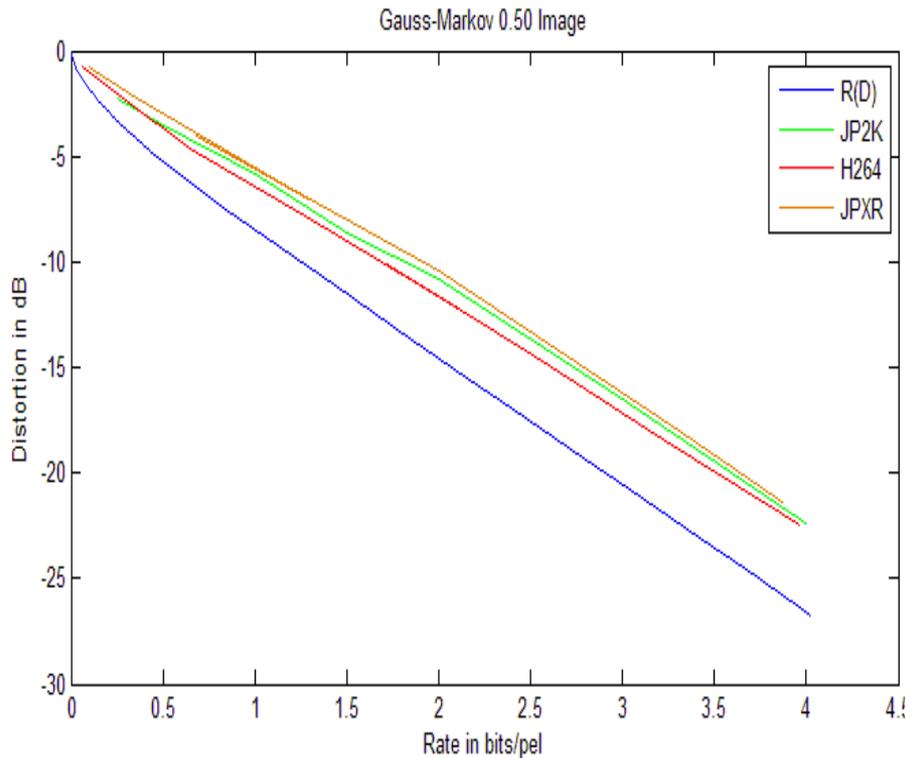
(Eigenvalues normalized for unit variance)

# Comparisons



$$\text{PSNR} = 22.11 - D \text{ dB}$$

# More Comparisons



$$\text{PSNR} = 22.11 - D \text{ dB}$$

# Lossless Compression

Correlation Parameter	Joint Entropy	Differences from Entropy (b/p)			
		SPIHT	CALIC	JP2K	JPEG-XR
0.95	3.0172	0.9438	0.4188	0.4838	0.9329
0.90	3.9778	0.6652	0.2312	0.3762	0.6847
0.50	5.9548	0.2392	0.0872	0.3172	0.2939
0	6.3691	0.2469	0.1779	0.3639	0.4420

\* CALIC closest to entropy in all cases

\* Aside from CALIC, SPIHT at  $a = 0.5$  and 0 beats others

# What Have We Learned?

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- Much room for improvement for lossy compression :
    - $> 0.5$  bpp for high quality
    - 4 to 6 dB at useful bit rates
  - Small room for improvement for lossless compression -  $\sim 0.2$  bpp
- \*\*Lesson: The best adaptive techniques can take you only so far.**

# Where to go from here?

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- For pure compression, much more potential payoff for lossy methods.
- Clearly advantageous to transform to independent variables and/or segment to stationary entities.
  - closes performance to the latter gaps
- Barring advancements in pure compression, need to pursue
  - better transforms that are adaptive to image features
    - Bandelets, curvelets, etc. ?
  - better segmentation and set partitioning methods

# Future Application Space

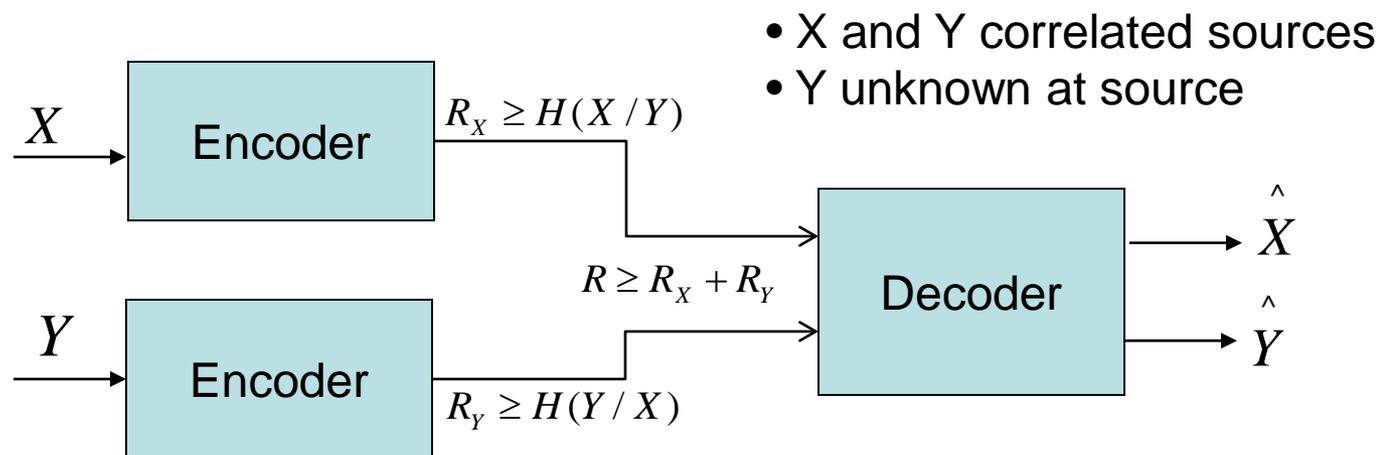
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Question: *Do we need more efficient compression?*

- Large images with multiple dimensions
  - Examples:
    - 4 dimensions: fMRI, medical ultrasound
    - Materials micro-structures with many attributes at given grid point.
- Content-based retrieval from large databases
  - Internet application needs interactivity for consultation and quantitative analysis.
  - Need fast search and retrieval and fast scalable decoding for browsing, retrieval, and transmission
    - Places limits on complexity and memory usage
      - Increase in size always seems to outpace gains in speed
    - Not likely to close existing performance gaps with simpler techniques that utilize less memory.
      - Fruitful or fruitless pursuit?
    - Contribution is to limit degradation the least possible by being clever
- ❖ Transmission rate decrease is main motivation for compression
  - Storage reduction now secondary

# Distributed Source Coding

Source Coding with Side Information: Slepian-Wolf 1973, Wyner-Ziv 1976



S-W: Encode X with  $H(X/Y)$  bits, Y with  $H(Y)$  bits, can achieve  $\hat{X} = X$   
*No loss over when Y is known at encoder also, if statistics X given Y are known.*

W-Z: *Lossy coding performance same whether Y is known at both ends or only at decoder, if statistics of X and Y are jointly Gaussian.*

# DSC Image Compression Scenarios

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- Low complexity encoding for image transmission
  - Sensor networks
    - Multiview coding
  - Multiple description coding
  - Camera alignment
  - Cryptogram compression
- ❖ None likely to bridge identified performance gaps, especially for the usual non-Gaussian lossy coding

# Technology Advances

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- Dramatic increases in processor speeds seem to be ending
  - Parallelization by multi-core processor chips is the trend
    - Power consumption  $\sim f^3$
  - New parallel forms of algorithms for compression likely to emerge
    - Currently JPEG2000, JPEG, etc. have parallel structure --- currently not exploited
    - Multiple description coding; distributed source coding
- More compact, higher power batteries would expand application scenarios for compression
- Miniaturization to quantum limit to be reached in 10 to 15 years
  - Quantum Computers: lower rate limits theoretically possible

# Quantum Computing

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- Quantum computers can solve some math problems considerably faster than classical computers
- Qbit.com (defunct) – claimed 2-10:1 lossless image compression at 1.5 Gbits/sec throughput
  - with qubit processor? US 2004/0086038 App.
- Quantum Information Theory
  - Well developed; parallels Shannon theory
    - Source coding theorem (von Neumann entropy limit)
    - R(D) theorem
    - S-W and W-Z theorems
    - Channel capacity theorem
  - Theoretically achievable rates lower than in classical computing

# Quantum Bits and Entanglement

- General state of one qubit (input):  $\alpha$  's complex

$$|\psi\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle, \quad |\alpha_0|^2 = \Pr\{|0\rangle\} \quad |\alpha_1|^2 = \Pr\{|1\rangle\}, \quad |\alpha_0|^2 + |\alpha_1|^2 = 1$$

- said to be *entangled*

Ex.: photon  $|\psi\rangle = (1/\sqrt{2})|0\rangle + (1/\sqrt{2})|1\rangle$  linear polarized at  $45^\circ$

- Output is measurement:  $|0\rangle$  or  $|1\rangle$ 
  - Orthogonal states can be measured
  - Similarly for 2-qubit system- states are *entangled*

$$|\psi\rangle = \alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle$$

- *n-qubit space –  $2^n$  dimensional Hilbert Space*
- *States can not be copied or cloned.*
- *A measurement changes the state: basis of secure key distribution*
- *States can be communicated*

# Entropy Example

Two equiprobable photon states: Shannon entropy = 1 bit

Suppose 0  $\rightarrow$   $|0\rangle$  H polarization

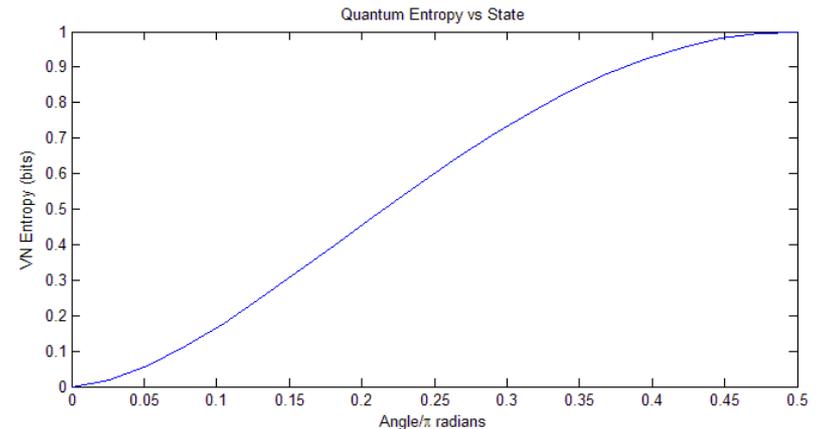
Suppose 1  $\rightarrow$   $\psi = \cos \theta |0\rangle + \sin \theta |1\rangle$  Angle  $\theta$  polarization

Von Neumann Entropy  $S(|0\rangle, \psi) = H_2((1 - \cos \theta) / 2)$

$$H_2(p) = -p \log_2 p - (1-p) \log_2 (1-p)$$

$$H_2(p) = -p \log_2 p - (1-p) \log_2 (1-p)$$

(Binary entropy function)



Except for  $\theta = \pm \pi / 2$  ,  $S(|0\rangle, \psi) < 1$  (e.g., 0.60 at  $\theta = \pi/4$ )

But, only  $\theta = \pm \pi / 2$  is detectable or communicable!!

❖ Therefore, von Neumann entropy not yet physically realizable.

# Prospect of Lower Compression Limit

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- So far, quantum information theory does not give physically realizable lower entropy limits
- Also, the devices and detectors work only in the laboratory or with limited capability – polarizers, 1-qubit gates, and short shift registers
- Short error-correcting codes, secure key distribution
- Physicists are hard at work to make the devices that form and detect specified quantum states
- Physicists have taken the lead at formulating quantum information theory, but IT community has been roused (e.g., Devetak & Berger, “Quantum R-D Theory,” Trans. IT Jun 2002; Rob Calderbank)
- Further reading
  - E. Desurvire, *Classical and Quantum Information Theory* (Cambridge 2009)
  - M. A. Nielsen, I. L. Chang: *Quantum Computation and Quantum Information*
  - N. D. Mermin : *Quantum Computer Science: An Introduction*
  - J. Audretsch, Ed.: *Entangled World: The Fascination of Quantum Information and Computation*
  - Bennett & Shor, “Quantum Information Theory”, Trans IT, Oct 1998

# Conclusion

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- Substantial gaps to compression limits still exist
- Trend toward algorithms working in small coding units and using complex entropy coding
- Trend to multiple core processors to spur development of new parallel processing paradigms
  - Collaborative compression
- Open question whether quantum information theory and quantum computation will bring future rate savings

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*Thank you!*